An investigation into the utilisation of Knudsen pumps in the operation of boundary-layer suction

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Abstract

This paper describes an investigation into combining two types of technology, Knudsen pumps and boundary-layer suction. It was the intention to use tried and tested modelling methods to demonstrate the potential benefits their integration can bring to the eco-efficient aviation industry.

Nomenclature

P	Pressure	Pa
T	Temperature	K
γ	Isentropic expansion factor	
Q_T	Temperature coefficient	
Q_P	Pressure coefficient	
δ	Rarefaction parameter	
a	Hydraulic radius	m
μ	Dynamic viscosity	kg/ms
m	Mass of gas molecule	kg
k	Boltzmann Constant	m^2kg/s^2K
Kn	Knudsen number	
λ	Mean free path	m
D	Collision diameter	m
y	Displacement perpendicular to surface	m
x	Displacement parallel to surface	m
u	Velocity component in x-direction	m/s
v	Velocity component in y direction	m/s
v_s	Suction velocity	m/s
U_f	Flight speed	m/s
x_{cr}	Transition point measured from leading edge	m
Re_{crit}	Critical Reynolds Number	
ρ	Density	kg/m^3
C_f	Skin friction drag coefficient	
C_d	Skin friction drag coefficient with suction	
δ_1	Displacement thickness	m
δ_2	Momentum thickness	m
P_w	Pump power input	W
s	Length of applied suction	M
l	Length of plate	m
b	Width of plate	m

1. Introduction

The Knudsen pump is a unique type of pump since it contains no moving parts and operates solely on the basis of a temperature gradient, rather than power input. It therefore has notable advantages in terms of maintenance, reliability and power consumption. Knudsen pumps generate a pressure difference through the effect of thermal transpiration. This is a rarefied gas phenomenon which arises when a gas contained in separate chambers is connected by a micro-channel (figure 1). Knudsen pumps are named after Martin Knudsen, a Danish physicist who specialised in molecular gas flow dynamics. It was in 1910 when Knudsen first proposed the idea of using the thermal transpiration effect for gas pumping. Despite this, it has only been in the last few decades where its application has become more prominently researched due to the advancements in micro-machining technology.

As for boundary-layer suction, this technology has been known and researched extensively for many years. It is a method which is used to control the flow over a surface by way of sucking in the passing air (figure 2). In 1948, Kay carried out a detailed experimental analysis on the effect of applying uniform suction over a flat plate [1]. This paper and many others have shown boundary-layer suction to preserve laminar flow, increase lift and improve velocity distributions. The method by which suction is generated is normally through the use of a vacuum pump. As far as research goes, there has been little or none on the study of incorporating Knudsen pumps into the application of boundary-layer suction. It is therefore the intention of this study to assess the feasibility of this proposal and lay the foundations for future work.

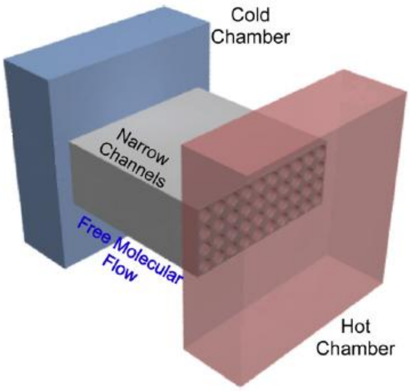

Figure 1 - Knudsen Pump

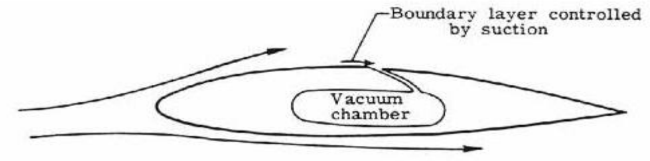

Figure 2 - Boundary-Layer Suction

2. Theory and Method

2.1 Knudsen Pump Modelling

When a fluid runs through a micro-channel, simple laws of flow cannot be applied to accurately model its characteristics. At this level, the flow becomes very sensitive to its surroundings and must be considered from a molecular point of view in order to effectively analyse its properties. The thermal transpiration effect in a Knudsen pump consists of two opposing flows, the Poiseuille flow caused by the pressure difference and the thermal creep flow caused by the temperature difference. There has been a range of analytical models developed to simulate this thermal transpiration effect, most famously the Direct Simulation Monte Carlo (DSMC). This method is highly comprehensive and adopts an approach based on probability to solve the Boltzmann equation. DMSC provides a benchmark by which other, more simplistic, thermal transpiration models are compared. A recent comparison of a selection of models found Sharipov's to be the most accurate when compared with

DSMC [2]. It is Sharipov's *s-model* which was adopted in this study due to its simplicity and certainty.

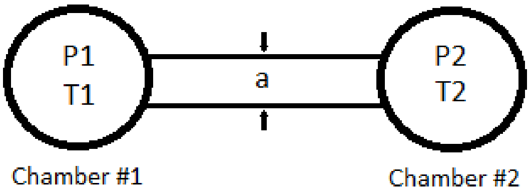

Figure 3 - Theoretical Knudsen Pump

Sharipov's specific study into rarefied gas flow through a long tube proved most relevant for the application of modelling a Knudsen pump [4]. Considering the system shown in figure 3, the relationship between pressure and temperature in each chamber is represented by equation 1:

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\Upsilon} \tag{1}$$

Flow is considered to be compressible and the isentropic expansion factor, Υ, is expressed as a function of Q_T and Q_P:

$$\Upsilon = \frac{Q_T}{Q_P} \tag{2}$$

Q_T and Q_P are the coefficients which describe the thermal creep and Poiseuille flows, respectively. Sharipov's method calculates these coefficients using discrete integration based on the bulk velocities and heat flux. These coefficients are largely dependent upon the rarefaction parameter, δ:

$$\delta = \left(\frac{aP}{\mu}\right)\left(\frac{m}{2kT}\right)^{1/2} \tag{3}$$

The Knudsen number is described by equation 4:

$$Kn = \frac{\lambda}{a} \tag{4}$$

The Knudsen number gives an indication on the type of flow regime.

Free molecular	$Kn > 10$
Transitional	$0.1 < Kn < 10$
Viscous	$0.01 < Kn < 0.1$

It is important that, when designing a Knudsen pump, the micro-channel only permits flow in the free molecular direction, and not transitional flow or viscous flow regimes. The significance of the mean free path is therefore apparent:

$$\lambda = \frac{kT}{\sqrt{2}\pi D^2 P} \tag{5}$$

Many studies of Knudsen pumps have focused on the application of a single gas [6, 4]. In this study an effective method of calculating the viscosity of dry air was adopted [7]. This method used equations developed from experimental data for the four major components of air: nitrogen, carbon dioxide, oxygen and argon. From this process, an approximation of the average collision diameter could be made.

The aim of the above equations were to determine the pressure in chamber #2 and hence the pressure difference. Once this was determined, it could be integrated with the pressure difference required from the boundary-layer suction.

Microsoft Excel was chosen as a platform for these calculations. To simplify the analytical process, tabulated data for Q_T and Q_P was outsourced and input to excel for each rarefaction parameter [4]. An online calculator on Sharipov's website provided a useful tool to verify the replicated model in Excel [5]. The following design parameters were used to alter the Knudsen pumps' performance in order to meet the required suction pressure:

- Temperature T_1
- Temperature T_2
- Hydraulic radius, a
- Capillary length, x

2.2 Boundary-Layer Suction Modelling

Since the purpose of this study was to investigate the feasibility of incorporating a Knudsen pump into boundary-layer suction, the assumption of a flat plate was deemed the simplest and most effective solution for analysing the boundary layer. The Blasius equation is used to describe the boundary-layer profile along a flat plate:

$$ff'' + 2f''' = 0 \tag{6}$$

In the case of a flat plate at zero incidence with uniform suction, the following set of differential equations has been shown to describe the system [8, 9]:

$$\frac{\partial y}{\partial x} + \frac{\partial y}{\partial x} = 0 \tag{7}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{d^2u}{dy^2} \tag{8}$$

Kay demonstrated that by applying the following boundary conditions, the velocity profile is characterised by an asymptotic exponential equation:

$$At\ y = 0;\ u = 0\ \ v = v_s = const$$

$$At\ y = \infty;\ u = U_f\ \ v = v_s$$

$$\frac{u}{U_f} = 1 - e^{\left({v_s y}/{\nu}\right)} \tag{9}$$

The main purpose of applying suction across a surface is to delay the transition from laminar to turbulent. If suction starts when the air has transitioned from laminar to turbulent, then no amount of suction will be able to prevent the boundary-layer from detaching and becoming fully turbulent. It was therefore essential that suction begins in the laminar region. The transition point for the chosen surface, without suction was calculated via equation 10.

$$x_{cr} = \frac{Re_{crit}\mu}{\rho U_f} \tag{10}$$

The critical Reynolds number at which transition occurs was taken to be 5x10^5. Assuming that the flow was incompressible, the free-stream conditions play a significant role in determining the amount of laminar flow across the surface. In order to appreciate the reduction in drag, equations for calculating the skin friction drag across a flat plate without suction were adopted. The addition of suction in the turbulent region will produce little improvement in drag however; in the laminar region the drag can be significantly reduced. The skin friction drag coefficient across the laminar section was calculated using equation 11:

$$C_f = \frac{1.328}{\sqrt{Re_{crit}}} \tag{11}$$

The method adopted for the calculation of the skin friction drag coefficient across a flat plate with suction has been outlined by Kay and Iglisch [1, 9]. This considers the total drag to be made up of the momentum drag in the wake, and the drag equivalent of the power required to restore the sucked air. It also considers the suction velocity and length of the applied suction.

$$C_D = \frac{v_s}{U_f}\left[\frac{1}{\xi}\frac{\delta_2}{\delta_1} + \frac{s}{l}\right] \tag{12}$$

Where

$$\xi = \left(\frac{v_s}{U_f}\right)^2 \frac{U_f l}{\nu} \tag{13}$$

$$\delta_1 = \frac{\nu}{-v_s}; \delta_2 = \frac{1}{2}\frac{\nu}{-v_s} \tag{14}$$

The parameters δ_1 and δ_2 are the displacement and momentum thicknesses respectively which describe the profile of the boundary-layer thickness along the plate. Finally the pump power required to provide the suction was calculated via equation 15:

$$P_W = \frac{1}{2}\rho {U_f}^3 s\left(\frac{v_s}{U_f}\right) \tag{15}$$

Since the Knudsen pump does not require a power input (assuming the temperature difference is sustainably sourced) then the above equations can be used to provide an approximation on the potential power savings that can be gained through the use of Knudsen pumps over conventional pumps.

Mathcad was the chosen software to conduct the boundary-layer suction analytical calculations due to its built-in Runge-Kutta method for solving ordinary differential equations.

2.3 Modelling Input Conditions

For each of the models, it was important that a set of input conditions that correspond to aircraft flight conditions was developed. The dimensions and performance parameters for the Airbus A380 were used as a benchmark for this study. Table 1 shows the various input conditions that were incorporated into the models. Various sources were used in the gathering of the data [10].

Table 1 - Input Conditions

Parameter	Value
Free stream velocity, U_f	260m/s
Cruise altitude	36kft
Temperature, T_o	217K
Pressure, P_o	22730Pa
Density, ρ_o	0.365kg/m^2
Dynamic viscosity, μ_o	1.43x10^{-5}Pa·s
Length of plate, l	6.6m
Width of plate, b	79.8m

The aim of this study was to use the input parameters to calculate the following values:

- Improvement in drag coefficient through the introduction of suction.
- Boundary layer profile along a flat plate
- Length of applied suction and transition point
- Required suction velocity and pressure values across laminar section
- Power savings gained through the selection of Knudsen pumps over conventional pumps
- Knudsen pump key dimensions and parameters

3. Results

For the flat plate without suction, the transition distance from the leading edge was calculated to be 0.076m, which suggested that, at the given flight speed, density and viscosity, approximately 1.2% of the flow over the wing is laminar. The corresponding skin friction drag coefficient was found to be 0.001878. The velocity distribution for the Blasius solution without suction and asymptotic suction profile is shown in figure 4:

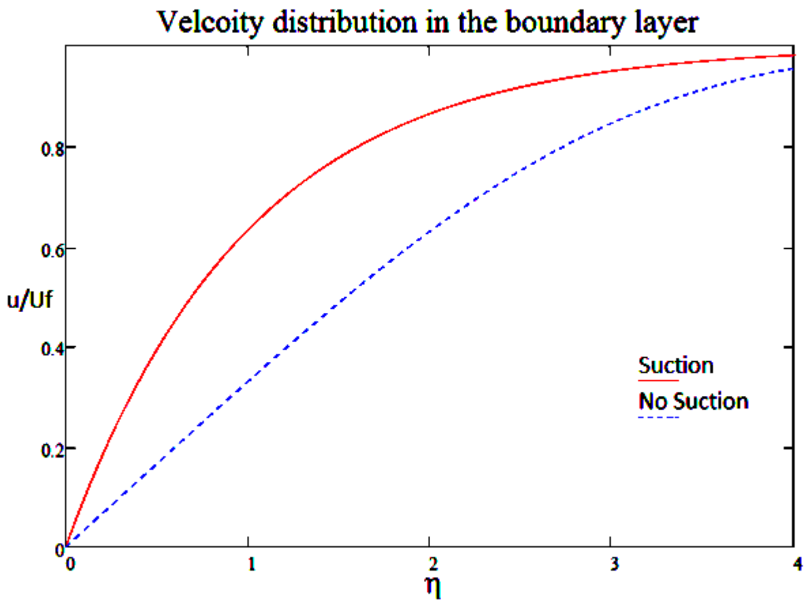

Figure 4 - Velocity profiles with and without suction

By setting the length of applied suction to be the same as the length of the laminar region (0.076m), and selecting a range of suction velocities, the change in drag coefficient could be observed in Figure 5:

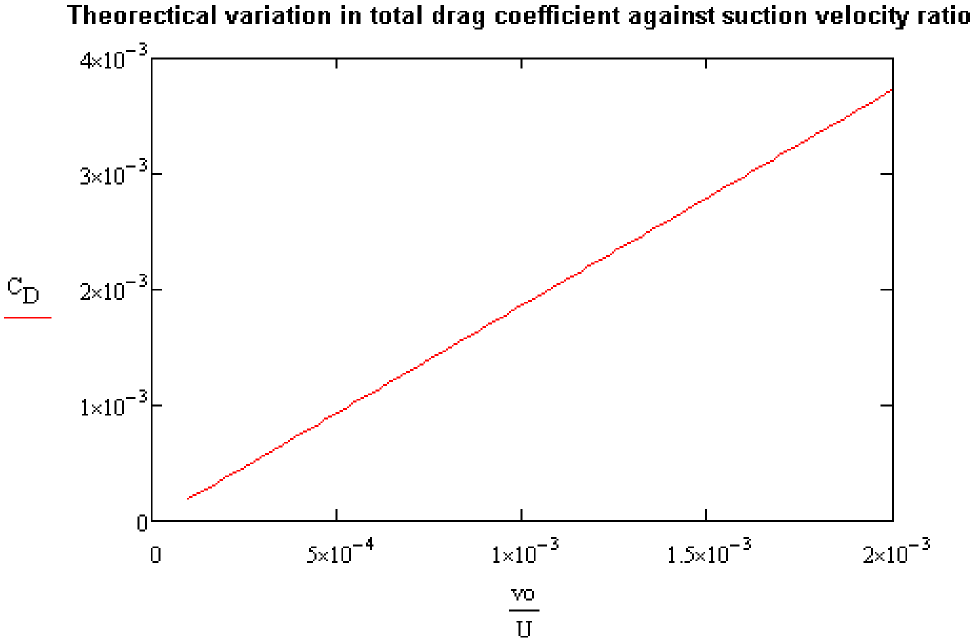

Figure 5 - Drag coefficient variation with suction velocity

A linear profile was observed that suggested the lower the suction velocity, the lower the drag coefficient. Considering the drag coefficient for the laminar section of the wing, a v_o/U ratio was chosen that would provide a significant reduction in $C_{d.}$ For v_o/U equal to 0.00077 (v_o=0.2m/s), the skin friction drag coefficient was observed to be 0.001438, a 23% improvement over the flat plate without suction. For the specified suction velocity, using standard equations the pressure difference required to provide the necessary suction was 3.3kPa (assuming the pressure at the inlet of the suction hole is equal to the ambient pressure). The pump power that would be required from a conventional pump was calculated to be 187W.

Taking this target pressure difference into the Knudsen pump model in Excel, a number of studies were carried out to determine a configuration that would allow this pressure difference to be met. How the pressure difference varied with each design parameter was studied in order to find an effective solution. Figure 6 and 7 show how the net pressure difference varied with the hydraulic radius and temperature difference. It was also observed that the pressure difference was independent of capillary length.

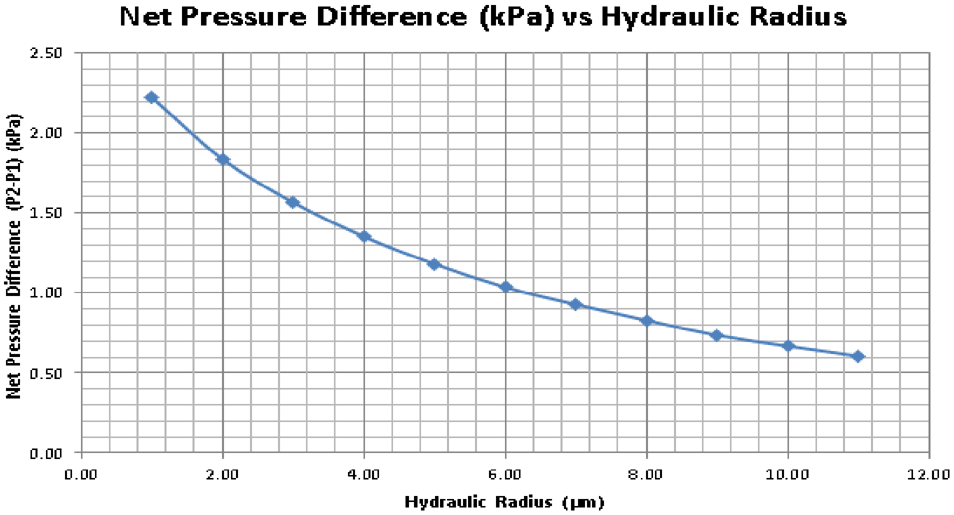

Figure 6 - Pressure difference vs. hydraulic radius

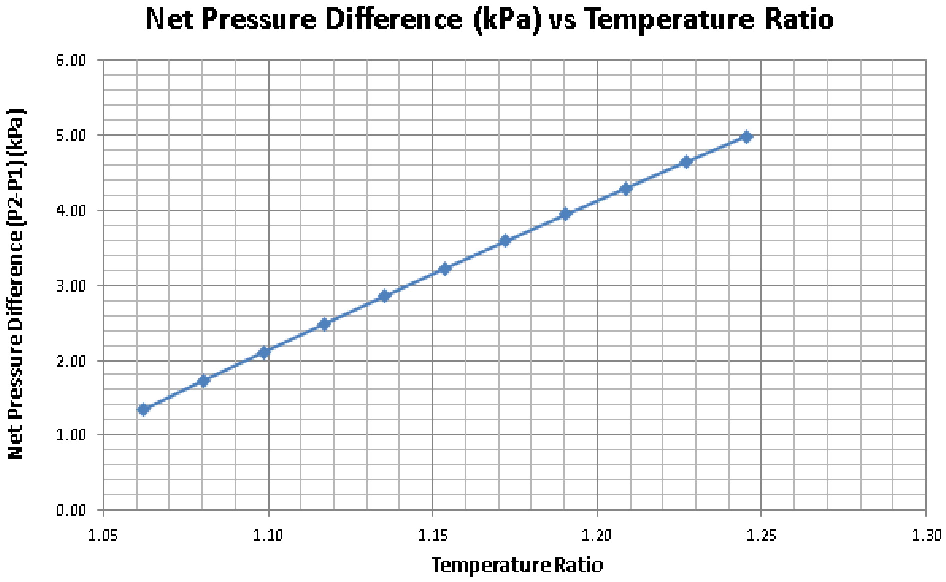

Figure 7 - Pressure difference vs. temperature ratio

From these graphs, it can be seen that a smaller hydraulic radius and larger temperature ratio produces larger net pressure differences. A temperature ratio and hydraulic radius of 1.28 and 1μm respectively resulted in a net pressure difference of 0.67kPa. In order to achieve the required pressure difference of

3.3kPa, a study of the Knudsen pump in series was carried out. Figure 8 shows an 11 stage Knudsen pump and the resulting pressure difference at each stage.

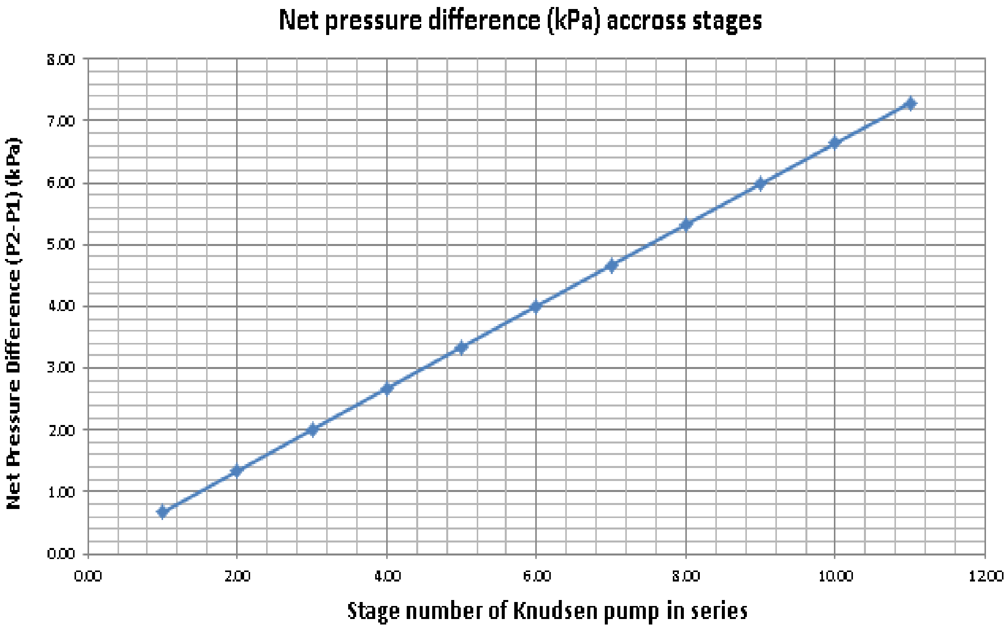

Figure 8 - Net pressure difference across Knudsen pump stages

It was observed that a 5-stage Knudsen pump connected in series would generate a pressure difference of 3.3kPa which is sufficient for the suction application.

A summary of the various design and output parameters for the Knudsen pump is shown in table 2 below.

Table 2- Knudsen pump design parameters

Design Parameter	Value
Temperature, T_1 (cold side)	273K
Temperature, T_2 (hot side)	350K
Pressure, P_1	22740Pa
Hydraulic radius, a	1µm
Capillary Length, L	50µm
Pressure, P_2	26083Pa
Pressure difference, ΔP	3.34kPa

4. Discussion and Conclusion

This study was able to successfully demonstrate through the use of analytical calculations that Knudsen pumps can be used in the operation of boundary layer suction. Boundary-layer calculations showed that, for a flat plate, the addition of suction can reduce the drag coefficient over the laminar part of the wing by 23%. Although the laminar part of the wing was only approximately 1.2% of the wings' average chord length without suction, the improvement is still considered to be significant when considering the scale of the Airbus A380's operations. In order to achieve this improvement in drag coefficient, a suction velocity of 0.2m/s was required. It should be noted that this is only relative to one suction hole; in practice many suction holes will be implemented. For the given suction conditions, a required pressure difference of 3kPa was calculated.

It was determined that a conventional pump would require 187W power input in order to provide the required suction pressure, however due to the nature of Knudsen pumps, this power input is not required and can therefore be used as an estimation of the potential power savings that can be gained, if Knudsen pumps are used instead of conventional pumps. By analysing how the net pressure difference varied with the design parameters of the Knudsen pump, an effective solution could be determined which met the required suction pressure. Theoretically Knudsen pumps can be connected in series to increase the pressure difference. In this study, a 5-stage configuration was observed to provide a net pressure difference of 3.3kPa.

From a theoretical point of view, the study was particularly successful in demonstrating that a Knudsen pump can be used in the operation of boundary layer suction. The results are encouraging in the sense of potential benefits and it leaves many additional possibilities for further improvement and research. A key point which is not elaborated in this study is how the temperature input into the Knudsen pump can be created. The aircraft de-icing or air conditioning systems are possible solutions to this problem which can be further researched. This study simply focused on the two dimensional flat plate theories for the boundary layer suction. A more accurate representation of the benefits could be achieved through the incorporation of a three-dimensional aerofoil.

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